METHOD OF MEASURING STATIONARY AND TRANSIENT THERMAL FLOWS

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UDC 536.24.083

A method is proposed for determining heat flux on the basis of experimental measurement of temperature as a function of time at two internal points of a flat wall.

A large number of investigations have recently been devoted to measuring and calculating heat flows. Many designs of calorimeters have been developed permitting the measurement of temperature as a function of time, with the result being used to calculate heat flux. Together with their advantages, present methods of measuring heat flux have certain shortcomings, which will be discussed briefly here. We will limit ourselves to nonstationary methods.

Horner's method* [1-3] is the simplest in the first group of methods to be discussed. The calorimeter here is a thin plate made of a material with known thermophysical properties. It is assumed that the temperature of the plate is independent of its thickness, i.e., the difference in temperature between the front and back ends may be ignored. Among the problems with this method is the need to use high-speed recorders and materials capable of withstanding high temperatures. Since the back end of the calorimeter is thermally insulated, the temperature of the plate increases very rapidly. Under high thermal loads, transducers of this type can be used only once [2].

The second group of methods are those that employ a quasi-stationary mode of heating by a heat flow acting on a plate insulated on one side. Included in this group are the methods of mean temperature [4], successive intervals [4], and the method presented in [5]. The shortcomings of these methods, as with the preceding method, include a short measurement time dictated by the maximum tolerable temperature of the transducer; advantages of these methods include the simple designs of the calorimeters, the need to record temperature at only one point on the transducer, and the simplicity of the formulas used to determine thermal flux.

With methods of yet a third group, the change in temperature over time is measured at two points on the transducer. The back wall of the transducer may be insulated or cooled [6-9]. The shortcomings of these methods include the complexity of the formulas for determining heat flux; advantages include the fact that the transducer can be cooled, so that measurement time is greater.

Below we propose a method which properly belongs to the third group, although the formula used to determine heat flux is fairly simple. The transducer in this method is a plate with known temperature dependences of its thermophysical properties. The plate is cooled on one side with water. A diagram of the transducer is shown in Fig. 1. The change in temperature over time is measured with thermocouples embedded in the plate at points with the coordinates x_1 and x_2. In order to determine the heat flux through the plate, we need to solve the equation of thermal conductivity

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a}\frac{\partial T}{\partial t} \tag{1}$$

with the boundary conditions

$$T(x, t)|_{x=x_1} = T_1(t), \tag{2}$$

$$T(x, t)|_{x=x_2} = T_2(t) \tag{3}$$

and initial condition

$$T(x, t)|_{t=0} = T_0 = 0. \tag{4}$$

*Known in the Soviet literature as the method of E. V. Kudryavtsev. Published in 1948. A similar reference was made in [4].

Cracow Polytechnic Institute, Polish People's Republic. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 38, No. 6, pp. 1017-1024, June, 1980. Original article submitted October 28, 1977.

0022-0841/80/3806-0629$07.50

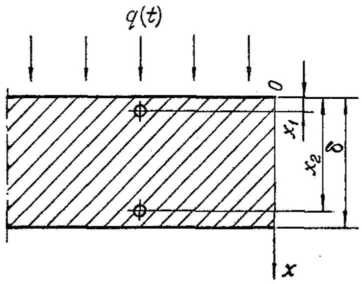

Fig. 1. Diagram of measurement plate: x_1, x_2) points at which thermocouples are embedded.

We will solve the above problem by using the method of functional corrections [10] as a first approximation. In accordance with this method, the local rate of change in temperature in Eq. (1) is replaced by the average rate

$$\frac{\partial^2 T}{\partial x^2} = f(t), \tag{5}$$

where

$$f(t) = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} \frac{1}{a} \frac{\partial T}{\partial t} dx = \frac{1}{a} \frac{\partial T_{av}}{\partial t}. \tag{6}$$

After integrating Eq. (5) twice with respect to x, we obtain

$$T = \frac{1}{2} f(t) x^2 + C_1(t) x + C_2(t). \tag{7}$$

Constants $C_1(t)$ and $C_2(t)$, determined from boundary conditions (2) and (3), have the form

$$C_1(t) = \frac{T_2(t) - T_1(t)}{x_2 - x_1} - \frac{1}{2} f(t)(x_1 + x_2) \tag{8}$$

and

$$C_2(t) = \frac{T_1(t) x_2 - T_2(t) x_1}{x_2 - x_1} + \frac{1}{2} f(t) x_1 x_2. \tag{9}$$

Taking Eqs. (8) and (9) into account, we write Eq. (7) in the form

$$T = \frac{1}{2} f(t) x^2 + \left[\frac{T_2(t) - T_1(t)}{x_2 - x_1} - \frac{1}{2} f(t)(x_1 + x_2) \right] x + \frac{1}{2} f(t) x_1 x_2 + \frac{T_1(t) x_2 - T_2(t) x_1}{x_2 - x_1}. \tag{10}$$

Substituting Eq. (10) in (6), we obtain the differential equation for determining f(t)

$$\frac{df(t)}{dt} + \frac{12a}{(x_2 - x_1)^2} f(t) = \frac{6}{(x_2 - x_1)^2} \frac{d}{dt} [T_1(t) + T_2(t)]. \tag{11}$$

Designating

$$p = \frac{12a}{(x_2 - x_1)^2}, \quad r = \frac{6}{(x_2 - x_1)^2} \frac{d}{dt} [T_1(t) + T_2(t)], \tag{12}$$

the solution to Eq. (11) may be written thus

$$f(t) = \left[C + \int_0^t r(t) \exp(pt)\, dt \right] \exp(-pt). \tag{13}$$

It follows from initial condition (4) that C = 0. Equations (10) and (13) determine the temperature field in the plate caused by the heat flowing through it, which can be computed from the formula

$$q(t) = -\lambda \left. \frac{\partial T}{\partial x} \right|_{x=0}. \tag{14}$$

Substituting Eq. (10) in (14), we obtain the formula for determining the measured heat flux

$$q(t) = -\lambda\left[\frac{T_2(t)-T_1(t)}{x_2-x_1} - \frac{1}{2}f(t)(x_2+x_1)\right]. \quad (15)$$

Taking Eq. (13) into account, we finally have

$$q(t) = -\lambda\left[\frac{T_2(t)-T_1(t)}{x_2-x_1} - \frac{1}{2}(x_2+x_1)\exp(-pt)\int_0^t r(t)\exp(pt)\right], \quad (16)$$

where p and r are determined from Eqs. (12).

At $t \to \infty$, Eq. (16) may be simplified, f(t) = 0, which follows from Eq. (6), and heat flux is determined from the formula

$$q = -\lambda\frac{T_2(t)-T_1(t)}{x_2-x_1}; \; q = \text{const}. \quad (17)$$

Equation (17) is well known [3,12] and is often used in measurements of the time constants of heat-flux probes with a water-cooled back wall.

For the case where the back wall of the probe is insulated and q = const, f(t) has the form

$$f(t)|_{t\to\infty} = \frac{1}{a}\frac{\partial T_{\text{av}}}{\partial t} = \frac{V_{\text{av}}}{a} = \text{const}. \quad (18)$$

Taking into consideration the fact that the problem was solved as a first approximation, the working formulas may be considered applicable for Fo $\geqslant 0.1$.

In certain cases, the method proposed here for measuring heat flux reduces to an already well known and widely used method. Let us analyze its relationship to Horner's method. To this end, we integrate Eq. (5) with respect to x from $x_1 = 0$ to $x_2 = \delta$ and obtain

$$\left.\frac{\partial T}{\partial x}\right|_{x=\delta} - \left.\frac{\partial T}{\partial x}\right|_{x=0} = \delta f. \quad (19)$$

Using Eq. (6) above, we arrive at the equation

$$\left.\frac{\partial T}{\partial x}\right|_{x=\delta} - \left.\frac{\partial T}{\partial x}\right|_{x=0} = \frac{1}{a}\int_0^\delta \frac{\partial T}{\partial t}dx. \quad (20)$$

Employing premises similar to those used in Horner's method:

$$\left.\frac{\partial T}{\partial x}\right|_{x=\delta} = 0, \; T_1 = T_2 = T_{\text{av}}(t), \quad (21)$$

finally, from Eq. (20) we have

$$q(t) = -\lambda\left.\frac{\partial T}{\partial x}\right|_{x=0} = c\rho\delta\frac{\partial T_{\text{av}}}{\partial t}; \; \text{Fo} \geqslant 0.1. \quad (22)$$

Thus, we obtain the same relation as was derived in [1-3]. Let us now discuss the relationship with the method presented in [5], as well as the methods of mean temperature and successive intervals. All of the methods examined here employ a quasi-stationary mode for the heating of a plate with an insulated back wall with a constant heat flux at Fo $\geqslant 0.5$. The temperature field in the quasi-stationary mode is described by the equation in [11]:

$$T = T_0 + \frac{q\delta}{\lambda}\left[\frac{at}{\delta^2} + \frac{1}{2}\left(\frac{x}{\delta}\right)^2 - \frac{x}{\delta} + \frac{1}{3}\right]; \; \text{Fo} \geqslant 0.5; \; q = \text{const}. \quad (23)$$

Differentiating Eq. (23) with respect to t, we obtain

$$\frac{\partial T}{\partial t} = \frac{qa}{\lambda\delta} = V = \text{const} \quad (24)$$

Substituting Eq. (24) in (20), we have

$$q = -\lambda\left.\frac{\partial T}{\partial x}\right|_{x=0} = c\rho\delta V = c\rho\delta\frac{\partial T}{\partial t}, \quad (25)$$

i.e., a formula which agrees with the relations in [4,5]. The precise relationship of the method presented here with that discussed above can be easily shown by substituting Eq. (23) in Eq. (16), from which then follows the identity of the right and left members of Eq. (16).

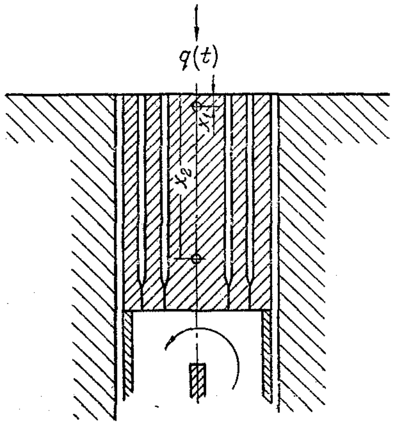

Fig. 2. Diagram of transducer for measuring thermal flux [3].

The above statements also apply to the method of successive intervals at q = const.

In conclusion, let us examine the method of mean temperature [4]. Since the back wall is insulated, Eq. (19) has the form

$$-\left.\frac{\partial T}{\partial x}\right|_{x=0}=\delta f. \tag{26}$$

Using the condition $\lambda(\partial T/\partial x|_{x=\delta}) = 0$, we will determine T_1 in conjunction with Eq. (10):

$$T_1=\frac{1}{2}f\delta^2+T_2. \tag{27}$$

Substituting Eq. (27) in Eq. (10), we have

$$T=\frac{1}{2}f(\delta-x)^2+T_2. \tag{28}$$

Considering that

$$T_{av}=\frac{1}{\delta}\int_0^\delta T dx=T_2+\frac{1}{6}f\delta^2, \tag{29}$$

we will determine the point where T, found from Eq. (28) with the coordinate x^*, is equal to the mean temperature at a given moment, i.e.,

$$T(x^*,\ t)=T_{av}(t). \tag{30}$$

Substituting Eqs. (28) and (29) in Eq. (30), we will have

$$\frac{x^*}{\delta}=1-\frac{\sqrt{3}}{3}\approx 0.4226. \tag{31}$$

Considering that $f = (1/a)(\partial T_{av}/\partial t)$ and taking Eq. (30) into account, we find from Eq. (26) that

$$q(t)=-\lambda\left.\frac{\partial T}{\partial x}\right|_{x=0}=c\rho\delta\frac{\partial T(x^*,\ t)}{\partial t}. \tag{32}$$

Thus, results are obtained that are identical to those obtained by the method of mean temperature. However, it should be noted that, in contrast to [4], there are no limitations on q.

There are several designs of transducer suitable for use in conjunction with the method proposed here, such as the designs presented in [3,12-17]. One of them is shown schematically in Fig. 2 [3]. Also, the calorimeters used in the methods discussed in this article can be employed.

We will examine use of the method proposed here using the example of a sudden increase in heat flux from zero to q_d = 200,000 W/m². The measurement transducer is a plate of thickness δ = 0.02 m, cooled with water at T_0 = 20°C. The plate is made of steel 20 and has a

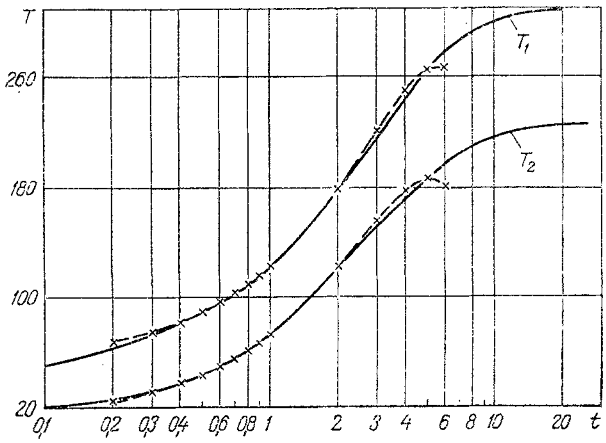

Fig. 3. Approximation of change in temperature of plate ($0 \leqslant x \leqslant \delta$) at points with coordinates $x_1 = 0$ and $x_2 = \delta$ with sudden heating of the plate surface by a constant heat flow [T, °C; t/33.9, a dimensionless number; t, sec].

coefficient of thermal conductivity $\lambda = 48.5$ W/m·°K. The coefficient of heat transfer from the back wall to the coolant water $\alpha = 970$ W/m^2·°K. The thermocouples were embedded in the plate on the back $x_1 = 0$ and front $x_2 = \delta$ walls. The change in plate temperature is shown in Fig. 3. Changes in temperature $T_1(t)$ and $T_2(t)$ [11] are approximated by polynomials

$$T_1 = a_0 + a_1 t + a_2 t^2, \quad 13.56 \leqslant t \leqslant 169.50 \text{ sec}, \tag{33}$$

where $a_0 = 52.12\,°C$; $a_1 = 2.275\,°C/\text{sec}$; $a_2 = -0.0060\,°C/\text{sec}^2$, and

$$T_2 = b_0 + b_1 t + b_2 t^2; \quad 13.56 \leqslant t \leqslant 169.50 \text{ sec}, \tag{34}$$

where $b_0 = 10.56°C$; $b_1 = 2.022°C/\text{sec}$; $b_2 = -0.0058°C/\text{sec}^2$.

The narrow time interval within which Eqs. (33) and (34) are valid is due to the approximation of T_1 and T_2 by quadratic polynomials. Equations (33) and (34) cannot sufficiently accurately describe the temperature throughout the entire interval to $t \geqslant 0.1\delta^2/a = 3.39$ sec.

Using a higher-order polynomial, we can make the approximation valid for a larger time interval. Substituting Eqs. (33) and (34) in Eq. (16), we obtain

$$q(t) = -\lambda \left\{ \frac{T_2(t) - T_1(t)}{\delta} + \frac{1}{2}\delta \left[\frac{1}{2a}(a_1 + b_1) + \frac{1}{a}\left(t - \frac{\delta^2}{12a}\right)(a_2 + b_2) \right] \right\}; \; 0.4\,\frac{\delta^2}{a} \leqslant t \leqslant 5\,\frac{\delta^2}{a}. \tag{35}$$

For time $t \to \infty$, $f \to \infty$ q(t) is determined by Eq. (17).

Table 1 shows values of q(t) determined from Eq. (35) in relation to time and the associated error. As can be seen from Table 1, the method is sufficiently accurate. It should be noted that the magnitude of the error is affected by the error of the approximation of the change in temperature over time (Fig. 2).

The exceptional simplicity of Eqs. (10) and (16) and the method of calculation based on the averaging of functional corrections follow from the above. From a practical point of view, the proposed method is preferable to most of the other methods, which lead to solutions in the form of infinite series using integral concepts and special functions (such as in [7-9]).

TABLE 1. Heat Flux Calculated from Eq. (35)

t, sec	13,56	16,95	20,34	23,73	27,12	30,51
q, W/m^2	192093	192477	192848	193209	193557	193893
R, %	—3,95	—3,76	—3,58	—3,40	—3,22	—3,05
t, sec	33,90	67,80	101,69	135,59	169,49	
q, W/m^2	194218	196818	198242	198588	197563	200014
R, %	—2,89	—1,59	—0,88	—0,71	—1,22	—0,01

For practical application of the proposed method, use may be made of existing calorimeter designs. Moreover, the measurement transducer does not require calibration.

NOTATION

$a = \lambda c\rho$, thermal diffusivity; a_0, a_1, a_2, coefficients in (33); b_0, b_1, b_2, coefficients in (34); $Bi = \alpha\delta/\lambda$, Biot number; c, heat capacity; C, constant; f(t), time functions in (6); $Fo = at/\delta^2$, Fourier number; q, heat flux; t, time; $T_1 = T(x_1, t)$, temperature at point x_1; $T_2 = T(x_2, t)$, temperature at point x_2; T_{av}, mean temperature; x_1, x_2, coordinates for embedment of thermocouples; x^*, coordinate of mean-temperature point; δ, thickness of plate; λ, thermal conductivity; ρ, density.

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